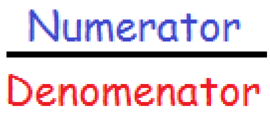
Fractions

The word 'fraction' comes from the Latin word for 'broken'. A fraction is a part of a whole, and can be displayed as two numbers separated by a line.



They can be expressed in different ways, for example: $\frac{1}{4}$ or $^{1}/_{4}$ or $1/4$.

A fraction consist of a numerator (the number on top or before the slash) and a denominator (the number on the bottom or after the slash).

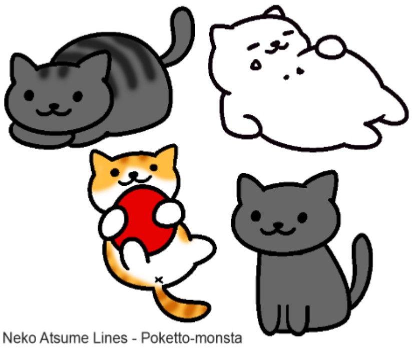


Figure 1: Litter of kittens.

Fractions are used to express a part of a whole. For example, if you have a litter of four kittens, you can think of this as four parts of one whole. In the litter (Figure 1), **two** kittens **out of a total of four** kittens are grey. This can be expressed as the fraction $\frac{2}{4}$.

In Figure 2, the same fraction is expressed in a circle. Two out of a possible four quarters of the circle, are coloured in blue.

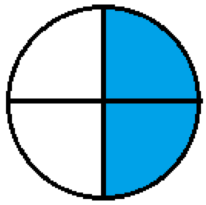


Figure 2: Circle divided into quarters.

The denominator can never be zero (0). This is because you cannot have parts of nothing. For example, if you have 0 cakes (meaning you have no cakes), 1 piece of that non-existent cake is still nothing. In fact, the whole concept of an imaginary cake is absurd. Therefore, the answer to 'something divided by nothing' does *not* equal zero, but instead it is undefined (essentially meaning it's nonsensical and cannot be determined).

Equivalent fractions

If you multiply the numerator and denominator of a fraction with the same number (not zero), you will get another fraction which is equal to the original fraction. For example, if we return to Figure 1 with the litter of kittens, we already determined that $\frac{2}{4}$ kittens are grey.

However, we could also express this as 'half the kittens in the litter are grey', or as the fraction $\frac{1}{2}$.

Illustrated by Figure 2 above, $\frac{2}{4}$ and $\frac{1}{2}$ are two ways of representing the same thing.

That is, two out of the four quarter circles are blue, and half of the circle is blue. By multiplying both the numerator and the denominator by 2, we can turn $\frac{1}{2}$ into $\frac{2}{4}$ (1×2=2, and 2×2=4).

Similarly, $\frac{10}{20}$, $\frac{25}{50}$, and $\frac{50}{100}$ are all different ways of expressing the same thing: half out of the whole, or $\frac{1}{2}$.

Simplifying fractions

As there are many different ways of expressing a fraction, it is usually simplified to the smallest numerator and denominator possible. For example, $\frac{1}{2}$ would be preferable over $\frac{2}{4}$. The process of finding the smallest possible numerator and denominator is called simplifying a fraction.

Activity

1) Simplify the fractions: a) $\frac{2}{10}$, b) $\frac{6}{42}$, c) $\frac{18}{81}$ (Refer to the end to check your answer)

Proper & improper fractions

In a **proper fraction**, the numerator is a smaller number than the denominator, for example $\frac{3}{4}$.

In an **improper fraction** (sometimes called a 'top heavy fraction'), the numerator is larger than the denominator, for example $\frac{7}{4}$.

Mixed numbers

A mixed number (also called a mixed numeral or mixed fraction) is the sum of a whole number, and a fraction. For example, $2 + \frac{3}{4} = 2\frac{3}{4}$.

Any improper fraction can also be expressed as a mixed number. For example, $\frac{7}{4}$ could be expressed as the mixed number $1\frac{3}{4}$.

Activity

2) Convert the following improper fractions into mixed numbers: a) $\frac{17}{5}$, b) $\frac{13}{4}$, c) $\frac{32}{5}$
3) Convert the following mixed numbers into improper fractions: a) $1\frac{1}{4}$, b) $3\frac{5}{7}$, c) $5\frac{2}{3}$

Adding & subtracting fractions

When adding fractions, it's important to remember that only *like quantities* can be added. This means that you can only add fractions that have the same denominator. If the denominator is the same, adding fractions is easy. You simply add the numerators together.

For example: If I want to add $\frac{2}{5} + \frac{1}{5}$, this is the same as $\frac{2+1}{5} = \frac{3}{5}$.

Adding fractions where the denominators are different, for example, $\frac{3}{4} + \frac{2}{3}$, can be a little more challenging.

In order to do this we need to convert the denominators to the same number. The way to do this, is to multiply each fraction with the lowest common denominator (LCD) between the fractions. The LCD is the lowest multiple; the lowest number that both denominators have in common. In this case, our denominators are 4 and 3. The multiple for these numbers is 12, and in this case, they do not have a lower multiple in common.In this case, we would multiply $\frac{3}{4}$ by 3, and $\frac{2}{3}$ by 4.

If we write it mathematically, it can look like this: $\frac{3}{4} + \frac{2}{3} = \frac{3\times3}{4\times3} + \frac{2\times4}{3\times4} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12}$.

If you are confused about the LCD, you can multiply the two denominators together for a common denominator. However, this means that in some cases you will be working with quite large numbers! After adding the fractions together, the final fraction can then be simplified to the lowest common denominator (see simplifying fractions on the previous page).

In this case, 17 is a prime number (meaning it can only be divided by itself or 1), hence the fraction cannot be simplified. We could write this fraction as a mixed number: $\frac{17}{12} = \frac{12}{12} + \frac{5}{12} = \mathbf{1\frac{5}{12}}$.

The method for subtracting fractions is the same as for addition, except the numerators are subtracted rather than added together.

Activity

4) Add the following fractions together. Simplify if appropriate and/or give the answer both as improper fractions and mixed numbers where appropriate.

   a) $\frac{1}{6} + \frac{2}{6}$ b) $\frac{8}{5} + \frac{2}{3}$ c) $\frac{3}{16} + \frac{4}{3}$

5) Subtract the following fractions. Simplify if appropriate and/or give the answer both as improper fractions and mixed numbers where appropriate.

   a) $\frac{3}{4} - \frac{1}{3}$ b) $\frac{6}{5} - \frac{2}{5}$ c) $\frac{4}{8} - \frac{4}{16}$

Multiplying fractions

When multiplying fractions, the numerators are multiplied together, and the denominators are multiplied together.

For example, $\frac{1}{2} \times \frac{2}{3} = \frac{1\times2}{2\times3} = \frac{2}{6}$ (which can then be simplified in to $\frac{1}{3}$).

The method is not difficult, however, why do we do this?

When multiplying fractions, we are essentially doing one fraction first, and then do the second fraction 'inside of' the first fraction.

To illustrate, half of the rectangle (Figure 3) is shaded pink (this is our $\frac{1}{2}$ fraction).

We can then further divide each half in to thirds. Out of our half, we then have two-thirds here shaded in green (this is our $\frac{2}{3}$ fraction).

The smaller rectangles shaded in green equal to two out of a total of 6 smaller rectangles (which also is one-third of the total shape!)

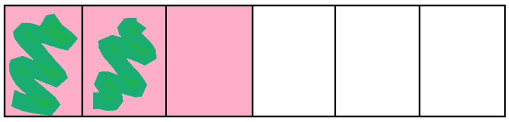


Figure 3: $\frac{1}{2} \times \frac{2}{3}$

We could do this the opposite way, that is, start with two-thirds of the shape. In figure 4, we start by colouring $\frac{2}{3}$'s of the shape pink.

Then we colour $\frac{1}{2}$ of the pink shapes green.

Our answer is still $\frac{2}{6}$ total shapes, which is equal to $\frac{1}{3}$ of the whole shape!

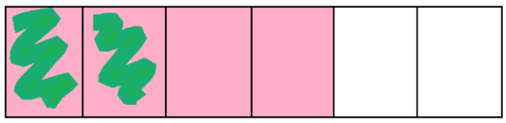


Figure 4: $\frac{2}{3} \times \frac{1}{2}$

Activity

6) Multiply the following fractions together

a) $\frac{1}{4} \times \frac{2}{3}$ b) $\frac{7}{2} \times \frac{1}{4}$ c) $\frac{2}{12} \times \frac{6}{7}$ (Refer to the end to check your answer)

Reciprocals

A **reciprocal** fraction is the number you need to multiply any given number with, in order to get the number 1 as a product. For example, the reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$, as $\frac{2}{1}$ multiplied with $\frac{1}{2}$ gives 1.

You can easily find the reciprocal fraction by inverting the fraction (making the numerator the denominator, and vice versa). You can think of this as 'flipping the numbers over'. This works with all fractions, so the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

Every number has a reciprocal except 0 (as discussed earlier, any fraction with 0 as a denominator is undefined).

Dividing by fractions

Dividing either a whole number or a fraction by a fraction is equal to the first number multiplied with the reciprocal of the divisor (the number that is dividing).

For example $3 \div \frac{1}{2}$ is the same as $3 \times \frac{2}{1}$.

Let's imagine that you and a friend bought three chocolate pralines. They are all different, and you both want a taste of each type of chocolate. What we want to do is divide the 3 chocolate blocks by half. This can be expressed mathematically as $3 \div \frac{1}{2}$.

So, you take a knife and you divide the chocolates in half. How many pieces of chocolate do you have now? Well, you have six pieces of chocolate! Hence, $3 \div \frac{1}{2} = 6$.

If you think about it, this is the same as 3 *multiplied* with 2, or $3 \times \frac{2}{1}$!

Activity

7) Divide the following fractions: a) $4 \div \frac{1}{2}$ b) $12 \div \frac{1}{4}$ c) $\frac{1}{3} \div \frac{1}{6}$

Answers

Note: The final answer is displayed in **red**

1) a) Divide both top and bottom by 2, to get $\frac{1}{5}$
   b) Divide both top and bottom by 6 to get $\frac{1}{7}$
   c) Divide top and bottom by 9 to get $\frac{2}{9}$
2) a) 5 goes in to 17 three times $(5 \times 3 = 15)$. Hence, we get 3 wholes.
   We then have $\frac{17}{5} - \frac{15}{5} = \frac{2}{5}$ left over. The answer is $3\frac{2}{5}$
   b) 4 goes in to 13 three times $(4 \times 3 = 12)$. Hence, we get 3 wholes.
   We then have $\frac{13}{4} - \frac{12}{4} = \frac{1}{4}$ left over. The answer is $3\frac{1}{4}$
   c) 5 goes in to 32 six times $(5 \times 6 = 30)$. Hence, we get 6 wholes.
   We then have $\frac{32}{5} - \frac{30}{5} = \frac{2}{5}$ left over. The answer is $6\frac{2}{5}$
3) a) $1\frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$ b) $3\frac{5}{7} = \frac{21}{7} + \frac{5}{7} = \frac{26}{7}$ c) $5\frac{2}{3} = \frac{15}{3} + \frac{2}{3} = \frac{17}{3}$
4) a) LCD=6. $\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$ which we could simplify to $\frac{1}{3}$
   b) LCD=15. $\frac{8}{5} + \frac{2}{3} = \frac{8\times3}{5\times3} + \frac{2\times5}{3\times5} = \frac{24}{15} + \frac{10}{15} = \frac{34}{15}$ which can be converted to the mixed number $2\frac{4}{15}$
   c) LCD=48. $\frac{3}{16} + \frac{4}{3} = \frac{3\times3}{16\times3} + \frac{4\times16}{3\times16} = \frac{9}{48} + \frac{64}{48} = \frac{73}{48}$ which can be converted to the mixed number $1\frac{25}{48}$
5) a) LCD=12. $\frac{3}{4} - \frac{1}{3} = \frac{3\times3}{4\times3} - \frac{1\times4}{3\times4} = \frac{9}{12} + \frac{4}{12} = \frac{13}{12}$ which can be converted to the mixed number $1\frac{1}{12}$
   b) $\frac{6}{5} - \frac{2}{5} = \frac{6-2}{5} = \frac{4}{5}$
   c) LCD=16. $\frac{4}{8} - \frac{4}{16} = \frac{4\times2}{8\times2} - \frac{4}{16} = \frac{8}{16} - \frac{4}{16} = \frac{4}{16}$ which can be simplified to $\frac{1}{4}$
6) a) $\frac{1}{4} \times \frac{2}{3} = \frac{1\times2}{4\times3} = \frac{2}{12}$ simplify to $\frac{1}{6}$
   b) $\frac{7}{2} \times \frac{1}{4} = \frac{7\times1}{2\times4} = \frac{7}{8}$
   c) $\frac{2}{12} \times \frac{6}{7} = \frac{2\times6}{12\times7} = \frac{12}{84}$ simplify to $\frac{1}{7}$
7) a) $4 \div \frac{1}{2} = 4 \times \frac{2}{1} = \frac{4\times2}{1} = 8$
   b) $12 \div \frac{1}{4} = 12 \times \frac{4}{1} = \frac{12\times4}{1} = 48$
   c) $\frac{1}{3} \div \frac{1}{6} = \frac{1}{3} \times \frac{6}{1} = \frac{6}{3} = 2$

Other helpsheets available

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- Percentages
- Rearranging Equations